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Question Paper Code : 91362

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2025.

Third Semester

Civil Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Aeronautical Engineering/Aerospace Engineering/Automobile Engineering/Biomedical Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich) /Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics /Petrochemical Engineering/Production Engineering/Robotics and Automation/Safety and Fire Engineering/Bio Technology/Biotechnology and Biochemical Engineering/Food Technology/Petrochemical technology/Petroleum Engineering/Pharmaceutical Technology)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Suppose that $z = \phi(x, y)$ and given a relation $z = (x^2 + a^2)(y^2 + b^2)$. Formulate a partial differential equation by eliminating the arbitrary constants a and b .
2. Find the complementary function for the PDE $(D^2 + 2DD' + D'^2)z = e^{x-y}$.
3. State Dirichlet's conditions for the existence of Fourier series. Does the Fourier series exist for a function where the function is discontinuous? Justify your answer.
4. What do you mean by harmonics? Why do we need Harmonic analysis?
5. Classify the PDE $x^2 u_{xx} + (1 - y^2) u_{yy} = 0$; $-\infty < x < \infty, -1 < y < 1$ as to whether it is elliptic, parabolic or hyperbolic type.
6. Find the steady state solution for the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(0) = 10$, $u(20) = 50$.
7. State of Fourier integral theorem.
8. State Parseval's identity for Fourier sine and cosine transforms.

9. Does the Z transform of $\{2^n\}$ exist everywhere in the whole complex plane? Justify your answer.
10. Formulate a difference equation by eliminating the arbitrary constants A and B from the relation $y_x = A(5^x) + B(7^x)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the singular solution of the partial differential equation

$$\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} - \sqrt{pq}. \quad (8)$$

- (ii) Find the general solution of the linear PDE

$$x(y^2 + z^2)p + y(z^2 + x^2)q = z(y^2 - x^2). \quad (8)$$

Or

- (b) (i) Find the particular integral for the PDE $(D^2 + 4DD' + 4D'^2)z = (e^{-2x} - e^y)^2$. (8)

- (ii) Solve $(D^3 - 7DD'^2 - 6D'^3)z = x^2 + 2x^2y + 2y^3$. (8)

12. (a) Obtain the Fourier series expansion of $f(x) = x^2 - x$ in $(-\pi, \pi)$. In addition sum the following series

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ and (8)

(ii) $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$ (8)

Or

- (b) Obtain the half range Fourier sine and cosine series expansion of

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & \text{if } \frac{\pi}{2} \leq x \leq \pi \end{cases} \quad \text{in } (0, \pi). \text{ In addition, sum the following series}$$

(i) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ and (8)

(ii) $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$ (8)

13. (a) A tightly stretched string of length 20 cm is fastened at both ends. The midpoint of the string is displaced by a distance 10 cm transversely and the string is released from rest in this position. Find the displacement of any point of the string at any subsequent time.

Or

- (b) An infinitely long metal plate in the form an area is enclosed between the lines $y = 0$ and $y = \pi$ for positive values of x . The temperature is zero along the edges $y = 0, y = \pi$ and the edge at infinity. If the edge $x = 0$ is kept at temperature ky , find the steady state temperature distribution in the plate.

14. (a) Find the Fourier Cosine transform of $\frac{1}{x^2 + a^2}$. Hence use suitable property, find the Fourier Sine Transform of $\frac{x}{x^2 + a^2}$.

Or

- (b) Find the Fourier Transform of $f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{otherwise} \end{cases}$, hence find the value

of $\int_0^{\infty} \frac{\sin ax}{x} dx$. Also find the function for which the Fourier Transform is $\frac{5 \sin 2(s - \pi)}{(s - \pi)}$.

15. (a) Find the Z Transform for the following

(i) $\{n\}$

(ii) $\{na^n\}$

(iii) $f(n) * g(n)$ if $f(n) = \left(\frac{1}{3}\right)^n$ and $g(n) = \cos n\pi$.

Or

- (b) Solve the difference equation $y(n+2) + 6y(n+1) + 9y(n) = n$, given that $y(0) = 0, y(1) = 1$.

